THEORETICAL AND EXPERIMENTAL RESEARCH ON VIBRATIONS OF THIN WALLED STIFFENED STRUCTURES

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Abstract

This paper deals with the problem of the natural modes of vibration in thin walled stiffened structures. A theoretical method was carried out which takes into account rigid displacements as well as distortion of the stringer cross section.

The results of the theoretical approach were compared with data obtained from testing stiffened panels; the tests were performed with an experimental apparatus which utilizes pneumatic exciters and displacement non-contacting transducers to obtain frequencies and modes of vibration.

Such a comparison shows a satisfying agreement between the theoretical and experimental results and emphasizes the importance of the cross section distortion as far as stress in the stringer is concerned.

Symbols

- A^L - action of the stringer relative to local displacements of a skin strip
- A^{SP} - action of the whole panel
- A^{ST} - action of the stringer relative to bending-torsion displacement
- A^{SK} - action of a skin strip
- A_S - cross area of the stringer
- b - width of the skin strip
- c - distance from stiffener centroid to skin cover mid-surface
- D - $\frac{Et^3}{12(1-\nu^2)}$, bending rigidity of the plate
- E - Young modulus
- f_e - experimental frequency, Hz
- $f_{B.T.}, f_L, f_{p.a.}$ - natural frequency computed respectively on the basis of stringer bending-torsion deformation, of local deformation and of present approach, Hz
- G - shear modulus
- L - lenght of panel
- m_l - number of longitudinal half-waves
- m_t - number of transversal half-waves between stiffener and edge of skin cover
- k^s, k^a - symmetrical and skew-symmetrical generalized deflectional stiffnesses defined by (2-1)
- T - kinetic energy
- $T^{T,L}$ - coupling term defined by (7)
- t - thickness of skin
- V - elastic potential energy
- $V^{T,L}$ - coupling term defined by (7')
- x_1 - rotation center distance from shear center in x coordinate
- y_1 - rotation center distance from shear center in y coordinate
- η - y/b
- θ_i - rotation of skin cover edge at the junction line with the i-th stiffener
- θ^L - rotation of junction line of stiffener skin strip
- θ^T - rotation of stringer cross section
- λ - $\frac{L}{m_l}$, longitudinal half-waves
- μ^F - generalized flange stiffness defined by (3-1)
- μ^L - generalized stiffness defined by (6)
- μ^s, μ^a - symmetrical and skew-symmetrical generalized rotational stiffness defined by (2-1)
- μ^T - generalized stiffness of stringer defined by (5)
- $(\mu k)^s, (\mu k)^a, (k\mu)^s, (k\mu)^a$ - coupling terms defined by (2-1)
- ν - Poisson coefficient
- ρ - density of material
- ξ_i - displacement of skin cover at the junction line with i-th stiffener
- ξ^L - displacement of stiffener skin strip in its own plane
- ω - circular frequency, rad/sec
- ω_i^* - $\sqrt{\frac{E\,t_i^2}{12(1-\nu^2)b_i^4}}$, reference frequency of i-th skin strip

Suffixes

- i - relative to i-th stiffener or skin cover strip
- ij - relative to i-th stiffener and to j-th junction line of i-th stiffener
- ijp - relative to i-th stiffener and to skin strip which joints j-th and p-th junction lines of i-th stinger

I. INTRODUCTION

Acoustic fatigue damage in aircraft structures basically depends on stress fluctuations excited by a random pressure field typically due to engine exhaust jets and/or aerodynamic turbulence. The evaluation of such stress is, therefore, a basic step in design against acoustic fatigue.

The structure stress response to a random pressure field can be suitably obtained by the normal mode approach; a basic step.

in such an approach, obviously, is the seeking of the frequencies and the normal modes of the involved structures.

As panels stiffened by an open section stringer are well representative of a large portion of those airplane structures which are able to endure acoustic fatigue damage, much attention is currently being paid to the problem of seeking the vibration modes of such panels.

This aim was, generally, pursued on the basis of an hypothesis of deformation which involves flexural displacement of the cover plate and bending-twisting of the stringer [1],[2]. On the contrary, the distortion of the stringer cross section due to inertia forces normal to its mid-surface was ignored. Such a distortion, however, can play an important role as far as the stringer stress resonant response is concerned, as the in service occurence of "acoustic fatigue crack in the bend radii of stringer" shows [3].

Methods have been, previously, developed to obtain modes and frequencies of vibration in stiffened panels taking into account only the stringer cross section distortion otherwise termed local deformation [4]. However only an approach which takes into account the interaction among bending, twisting and local deformations of the stringer cross section, can give, it is felt, a sufficiently accurate description of the stress state as far as acoustic fatigue problems are concerned.

The purpose of the present investigation is therefore to evaluate the influence of such an interaction on the modes of vibration of stiffened panels, tackling the question from both the theoretical and experimental point of view.

The theoretical approach was based on Hamilton's variational principle. The panel was considered simply supported at the edges orthogonal to the stringer axes; in such conditions its displacements can be, "exactly", expressed as functions of a finite number n of generalized coordinates q (namely the displacements and the rotations at each stringer-sheet cover junction line and at each boundary edge plus the rotation and the displacements of each stringer component flats at their edge lines, all these quantities being computed at a reference panel cross section).

With reference to such displacements the action can be reduced to a quadratic form in the coordinates q_i whose coefficients can be computed for each given geometry by a general procedure developed ad hoc. A set of linear homogeneous equations in the unknown q_i is then obtained through the application of the variational principle. The eigenvalue problem associated with such equations is then resolved to give the frequencies and the modes of vibration for each chosen panel.

At the same time tests were performed to obtain the frequencies and the modes of vibration of stiffened panels. The basic components of the testing carried out for such investigations are the following ones: a stiff rectangular frame for restraining the specimen, a set of eight sinusoidal pneumatic exciters, movable non-contacting transducers and an instrument chain which records the displacement amplitude and the displacement-force phase angle as functions of the excitation frequency. The frame was designed to obtain simply supported edge boundary conditions; the choice of the pneumatic exciters and non-contacting transducers allowed the change of inertial characteristics of the specimen to be minimized lastly the response both in amplitude and in phase allowed an unambiguous identification of the resonant conditions of the specimen.

The panels were made of a rectangular sheet-cover stiffened by only one Zed section stringer, as the choice of such a configuration gave the opportunity of displaying all the revelant features of the phenomenon under investigation. At the same time the reduced number of degrees of freedom of such specimens allowed a reduced number of exciters to obtain in an accurate way all the relevant modes of vibration of interest in the field of acoustic fatigue.

The experimental results were obtained in the form of frequencies and modes of vibration of the involved panel; they were used to control the accuracy of the proposed theoretical approach. The comparison shows a satisfying agreement between the theoretical and experimental results and demonstrates, it is felt, the soundness of the proposed theoretical approach. The results as a whole emphasize the importance of the stringer cross section distortion as far as the modes of vibration and therefore the stresses are concerned.

II. THEORETICAL APPROACH

The stiffened panel considered in the present investigation is made of a thin rectangular plate (sheet cover) reinforced by parallel open section stringers restrained to the plate along an edge. The cross section of a single stringer can be regarded as made up of a number of thin flat strips (component flats) joined edge to edge.

The frequencies and the modes of vibration of such structures can be sought through Hamilton's principles. The action:

$$A = \int_0^{2\pi/\omega} (T - V)\, dt$$

can be specified on the basis of the displacements of the panel mid-surface and the circular frequency of the vibration.

The mid-surface displacement amplitude of a panel vibrating in a natural mode is bounded to vary according to m_1 sinusoidal half waves along each line parallel to the stringer axes. Such a condition stems from the boundary conditions at the panel edges orthogonal to the stringer axes; therefore only the cross section displacements are required to be determined.

To this end consider at first a sheet cover strip between the i-th and (i+1)-th stringers, (i=1,2,3,...n); by letting the displacement and the rotation at the i-th stringer panel junction line be denoted by ξ_i and θ_i, the displacements of the strip mid-plane can be put:

$$w_i(x,z,t) = \left(\frac{\xi_i+\xi_{i+1}}{2} f_i^s(x) + \frac{\xi_i-\xi_{i+1}}{2} f_i^a(x) + \frac{\theta_i-\theta_{i+1}}{2}\phi_i^s(x) + \frac{\theta_i+\theta_{i+1}}{2}\phi_i^a(x)\right)\sin\left(\frac{\pi z}{\lambda}\right)$$

$$\sin(\omega t) = W_i(x)\, g(z,t) \qquad (1)$$

The functions f_i^s, f_i^a, ϕ_i^s, ϕ_i^a sketched in fig.1, once multiplied by $g(z,t)=\sin(\pi z/\lambda)\sin(\omega t)$, are particular solutions of the differential equation of the vibrating plate:

$$\nabla^4 w_i + \frac{\rho}{D}\frac{\partial^2 w_i}{\partial t^2} = 0 \qquad (2)$$

relative to the boundary conditions specified in fig. 1.

Owing to the property of symmetry of s functions and the skew-symmetry of the a functions and the symmetry of the domain of integration the Lagrangian is made up of only six terms: namely the Lagrangian relative to each function and two cross coupling terms coming from double products of the two symmetric and the two skew-symmetric terms.

Taking then the interval of integration equal to the period of the vibration, the action A^{SK} becomes therefore, following the notations defined by (1):

$$A^{SK}(w_i) = \left(\frac{\xi_i+\xi_{i+1}}{2}\right)^2 A(f_i^s g) + \left(\frac{\xi_i-\xi_{i+1}}{2}\right)^2 A(f_i^a g) + \left(\frac{\theta_i-\theta_{i+1}}{2}\right)^2 A(\phi_i^s g) + \left(\frac{\theta_i+\theta_{i+1}}{2}\right)^2 A(\phi_i^a g) +$$

$$+\left(\frac{\xi_i+\xi_{i+1}}{2}\right)\left(\frac{\theta_i-\theta_{i+1}}{2}\right) I^s(f_i^s g, \phi_i^s g) + \left(\frac{\xi_i-\xi_{i+1}}{2}\right)\left(\frac{\theta_i+\theta_{i+1}}{2}\right) I^a(f_i^a g, \phi_i^a g) \qquad (3)$$

where I^s and I^a are two operators which take into account the interaction terms.

Applying Hamilton's principle to the i-th strip deformed as previously specified, the action $A^{SK}(w_i)$ can be obtained as work done by the generalized edge forces in a period. Therefore following the sketch and symbols of fig.1, as the amplitude of the edge generalized displacements are unitary one obtains:

$$A(f_i^s g) = -\frac{\pi\lambda}{2\omega}\, 2\, k_i^s \quad ; \quad A(f_i^a g) = -\frac{\pi\lambda}{2\omega}\, 2\, k_i^a$$

$$A(\phi_i^s g) = -\frac{\pi\lambda}{2\omega}\, 2\, \mu_i^s \quad ; \quad A(\phi_i^a g) = -\frac{\pi\lambda}{2\omega}\, \mu_i^a$$

$$I^s(f_i^s g, \phi_i^s g) = -\frac{\pi\lambda}{2\omega}\, 2(\mu k)_i^s = -\frac{\pi\lambda}{2\omega}\, 2(k\mu)_i^s \quad ;$$

$$I^a(f_i^a g, \phi_i^a g) = -\frac{\pi\lambda}{2\omega}\, 2(\mu k)^a = -\frac{\pi\lambda}{2\omega}\, 2(k\mu)^a \qquad (3')$$

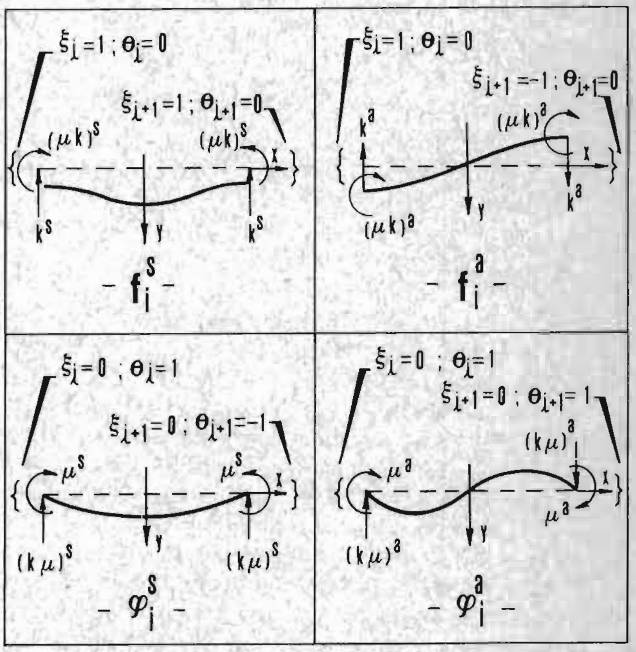

Fig.1- Basic deflection shapes of strip cross section for unit value of the related generalized coordinate. For each case the figure also shows the amplitudes of the force and the couple per unit length acting at each edge.

Therefore the computation of the action for the i-th strip is led back to the calculation of the generalized rotational and deflectional stiffnesses m_i^s, μ_i^a, k_i^s, k_i^a and the two cross coupling stiffnesses

$(\mu k)^s=(k\mu)^s$, and $(\mu k)^a=(k\mu)^a$ which in turn can be easily obtained as bending moments and shearing forces at the strip edges for the appropriate strip deflection. This procedure is much more time saving than the direct one which involves the computation of the elastic and cinetic energies and is quite general. Each stiffness is a transcendental function of the two quantities:

$$(\alpha_1)_i=\sqrt{\left(\frac{\pi b_i}{\lambda}\right)^2+\frac{\omega}{\omega_i^*}}\ ;\quad (\alpha_2)_i=\sqrt{\left(\frac{\pi b_i}{\lambda}\right)^2-\frac{\omega}{\omega_i^*}} \tag{4}$$

which involve the circular frequency of vibration ω, the strip width b_i, the longitudinal halfwave lenght $\lambda=L/m_1$ and the reference circular frequency

$$\omega_i^*=\sqrt{\frac{E}{12(1-\nu^2)}\ \frac{t_i^2}{b_i^4\rho}} \tag{4'}$$

The detailed expression of such stiffnesses are reported in appendix 1.

Let us now consider the i-th stringer displacements; suppose at first that the cross section is bounded to rotate as a whole around an axis of an angle $\theta_i=\theta_i^T \sin(\frac{\pi z}{\lambda})\sin(\omega t)$. The action can be put in the following form,[5] :

$$A_i^{ST}=\frac{\pi\lambda}{2\omega}\,\theta_i^T\Big\{\rho\frac{\omega^2}{2}\Big[I_0+\left(\frac{\pi}{\lambda}\right)^2(\Gamma_E+x_1^2 I_{xx}+y_1^2 I_{yy}-2x_1y_1 I_{xy})\Big]-\frac{1}{2}\Big[\left(\frac{\pi}{\lambda}\right)^2 GJ+E\left(\frac{\pi}{\lambda}\right)^4(\Gamma_E+x_1^2 I_{xx}+y_1^2 I_{yy}-2x_1y_1 I_{xy})\Big]\Big\}=\frac{\pi\lambda}{2\omega}\ \frac{1}{2}\,(\theta_i^T)^2\,\mu_i^T \tag{5}$$

where I_0 is the polar moment of inertia of the cross section about the rotation center, Γ_E is the warping constant relative to shear center E, $I_{xx}^{(*)}$, I_{yy}, I_{xy} are the moments and the products of inertia with respect to the x- and y- axes through the centroid, J the uniform torsion constant.

In the case of a stringer attached to the sheet, the axis of rotation is practically bound to lie in the mid-plane of the sheet so that y_1 is known; and x_1 is easily expressible in terms of θ_i^T and the displacement of the stringer sheet junction line ξ_i.

Suppose now that only a distortion of the stringer cross section is allowed. A complete description of such a stringer cross section distortion would require three generalized coordinates (two displacements and a rotation) for each edge line where component flats converge.

If the stiffness of a component flat to translation in its own plane is much greater than the stiffness to rotation, a situation which often verifies in such structures of interest in the acoustic fatigue, an approximate allowance for such distortion can be made assuming that no component flat is translated in its own plane during vibration and therefore the junction between flats remains fixed in space. In such hypothesis the distortion of the cross section consists of the pivoting of the component flats about their edge lines.

Let the angle of rotation about each edge line be denoted by $\theta_{i,k}^L$, where k=1,2...r, r being the number of stringer edges numbered starting from the stringer-sheet junction line.

Then the action results a quadratic form in the coordinates θ_{ik}^L of the following form:

$$A_i^L=\frac{\pi\lambda}{2\omega}\sum_{j=1}^{r}\sum_{k=1}^{r} a_{ijk}\,\theta_{ij}^L\theta_{ik}^L=\frac{\pi\lambda}{2\omega}\,\frac{1}{2}\mu_i^L(\theta_i^L)^2$$

$$\mu_i^L=2\frac{|a_{ijk}|}{M_{i11}}\ ;\quad \theta_{i1}^L=\theta_i^L \tag{6}$$

in which $\{a_{ijk}\}$ is a matrix symmetric with respect to the indexes j,k, $|a_{ijk}|$ is the matrix determinant, M_{i11} is the minor determinant corresponding to the element a_{i11}. The a_{ijk} coefficients have the following expression, [4]:

$$a_{ijj}=\sum_{s=1}^{q}\mu_{ijj}^F+\frac{1}{2}\sum_{p=1}^{r}(\mu_{ijp}^s+\mu_{ijp}^a)$$

$$a_{ijk}=\frac{1}{2}(\mu_{ijk}^s-\mu_{ijk}^a)\quad j\neq k \tag{6'}$$

where μ_{ijp}^s, μ_{ijp}^a are generalized rotational stiffnesses, already defined for sheet strips, at the j-th edge relative to the component flat which connects the j-th and p-th edge(*), μ_{ij}^F are the rotational stiffnesses of the flanges converging at the j-th edge. Also μ^F is a transcendental function of the same kind of μ^s and μ^a; its expression is also given in appendix 1.

The proposed model of the stringer cross section distortion, a model well known to people interested in stiffened panel local buckling stress calculation, can be used with confidence for typical ranges of panel dimensions which are encountered in air

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(*) Because of stringer bending the axial displacement of the attached flange is communicated to the sheet cover. In consequence I_{xx} must be increased by the addition of the term $\eta A_s b_i t_i^2 c^2/(A_s+\eta b_i t_i)$ where η is a factor lower than unity which represents the effect of the shear lag in the sheet-cover.

(*) If no flat connects the i-th and p-th edge such stiffnesses must be put equal to zero.

craft construction at least in the frequency range which broadly interests the acoustic fatigue.

Nevertheless if some component flat has such dimensions that it becomes the "driving element" in the vibration of the entire panel for frequency which lies in the field of interest the above idealization of the stringer distorted cross section can cause not negligible errors in frequency and mode shape calculations.

In such conditions the hypothesis that the edge lines at the junction between the flat remain fixed in the space cannot be completely retained even if it is generally unnecessary to release all the translational degree of freedom of the edge line. In fact the contribution to the action of several translational degrees of freedom results still negligible in the specified frequency field.

The choice of the translational degrees of freedom to be released must be done case by case on the basis of an accurate examination of the panel geometry and of the natural frequencies of each component flat considered isolated. Having chosen the translational degrees of freedom the action of the stringer can be computed summing up the contribution of each component flat which can be treated in the same way as the sheet-cover strip.

When both rotation and distorsion of the stringer cross section are allowed the action is the sum of the corresponding quantities relative to each type of deformation plus two groups of cross coupling terms one coming from the kinetic energy, the other from the elastic energy. Suppose at first that cross section distortion is allowed by rotational degrees of freedom and consider the kinetic energy of the coupled mode: in each component flat the displacement systems associated with torsion-bending modes of vibration are all virtually constant through the thickness. On the contrary the displacement systems associated with local modes of vibration are of two types, the displacement component orthogonal to the flat mid-plane being constant through the thickness,while the other two components linearly vary through the wall thickness with a mean value of zero. In consequence among all the double products coming from squaring each component of the resultant displacement velocity only the one relative to the displacements orthogonal to the flat mid-plane, once integrated through the thickness, results non zero.

Such a term, can be put in the form:

$$T^{T,L} = \frac{\pi\lambda\omega^2\rho t}{2\omega} \int_s w^L w_n^{BT} \, ds \qquad (7)$$

where w^L is the local displacement and w_n^{BT} is the bending-torsion displacement orthonal to the stringer cross mean line s.

The same expression holds when translational degrees of freedom are allowed in cross section distorsion; however in such a case other cross coupling terms arise from bending-torsion displacement velocity and the in plane local displacement velocity of component flats due to translational degrees of freedom. Such terms can be easily computed case by case once the translational degrees of freedom have been specified.

A cross coupling term which derives from the elastic potential energy is connected with the St.Venant torsional shear stresses which couple with the shear stresses due to the deflection of each component flat. Such a term can be put in the following form :

$$V^{T,L} = \frac{\pi\lambda}{2\omega}\left(\frac{\pi}{\lambda}\right)^2 \frac{G}{3} \sum_r t_r^3 \int_{f_r} \frac{\partial w^L}{\partial s} ds \qquad (7')$$

where t_r is the thickness of the r-th flat f_r, w^L the local displacements, s the abscissa along the flat length. If the stringer cross section distorsion depends only on rotational degrees of freedom no other cross coupling term can derive from the elastic energy. In fact the other stress systems associated with torsion-bending modes are constant through the thickness wall of each component flat, whereas the stress systems associated with local modes of vibration all vary linearly through the wall thickness and have a value of zero. Otherwise if translational degrees of freedom are allowed further cross coupling terms arise due to the stress systems constant through the thickness of component flats associated with their local in-plane displacements. Their computation is straightforward once the degrees of freedom have been chosen. The cross coupling terms were examined with some detail in ref. [6] where an appraisal of their influence on the frequencies and modes of vibration of the stiffened panels is made. At this stage it is possible to write the action for the whole stiffened panel. In fact for a stiffened panel made of n+1 strips and n stringer the relationships 3), 5), 6), 7), 7'),in the case of cross section distortion allowed only through rotational degrees of freedom yield the following expression for the action A^{SP} if an irrelevant factor $\frac{\pi\lambda}{4\omega}$ is removed:

$$A^{SP} = -\sum_{i=o}^{n} \left[(\xi_i + \xi_{i+1})^2 k_i^s + (\xi_i - \xi_{i+1})^2 k_i^a + (\theta_i - \right.$$

$$-\theta_{i+1})^2 \mu_i^s+(\theta_i+\theta_{i+1})^2 \mu_i^a+(\xi_i+\xi_{i+1})(\theta_i-\theta_{i+1})$$

$$(\mu k)_i^s+(\xi_i-\xi_{i+1})(\theta_i+\theta_{i+1})(\mu k)_i^a\Big]+\sum_{i=1}^{n}\Big[\mu_i^T(\theta_i^T)^2-$$

$$-\mu_i^L(\theta_i^L)^2 +T_i^{T,L}\theta_i^T\theta_i^L-V_i^{T,L}\theta_i^T\theta_i^L\Big]+(h_o\xi_o^2+l_o\theta_o^2+$$

$$+h_{n+1}\xi_{n+1}^2+l_{n+1}\theta_{n+1}^2)\Big] \qquad (8)$$

where $T_i^{T,L}$, $V_i^{T,L}$ are the cross coupling terms computed through relations 7) and 7'); the terms in the last bracket take into account boundary conditions of elastically supported and elastically built-in-edges at the panel side parallel to the stringer axes.

If translational degrees of freedom are allowed in the stringer cross section distortion the above expression must be modified as far as μ_i^L and the stringer cross coupling terms are concerned. This can be done straightforwardly on the basis of the preceding reasoning; in particular the contribution of each component flat whose edges translate and rotate can be computed on the basis of the procedure outlined for the sheet-cover strip.

Once the action has been written, differentiating with respect to each generalized coordinate and equating to zero, one obtains a set of linear homogeneous equations. The solution of the associated eigenvalue problem furnishes the frequencies and the modes of vibration of the stiffened panel.

An application of such a method to panels stiffened by one Zed section stringer is discussed in the following section in conjunction with a set of experimental data on the frequencies and modes of vibration of such panels obtained with the procedure outlined in the following section.

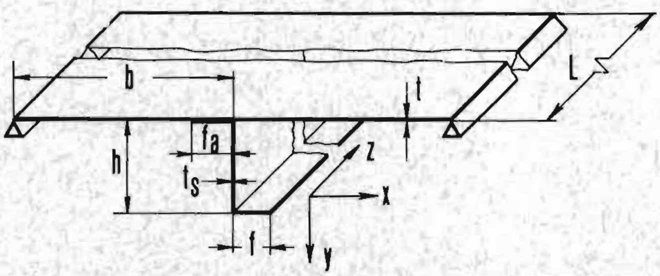
The solution of the above-said eigenvalue problem was also used to discuss the influence of some relevant geometric ratios on the vibratory behaviour of such panels. Other detailed theoretical results were also given in ref. [6].

III. EXPERIMENTAL RESULTS AND THEORETICAL APPROACH EVALUATION

The evaluation of the proposed theoretical approach was performed through a suitable set of data obtained by testing Zed section stringer stiffened panels in a test rig designed and built for this purpose.

The test facility was already described with some detail in ref. [7]; basically a rectangular stiff frame, joined to a woodden mount by low stiffness springs, restrained the stiffened panel furnishing simply supported edge boundary conditions. The panel excitation was obtained with sinusoidal pneumatic shakers which can be driven by a variable speed d.c.motor in the frequency range 20÷1000 Hz. The displacements were measured by a non-contacting capacitance-type transducer. Such a kind of shaker and transducer was chosen to make negligible the influence of the so called added masses on the dynamic behaviour of such a type of structure characterized by walls of small thickness. The displacement amplitude and the force-displacement phase angle could be continuously recorded, through a suitable instrument system, on a x, y, y', recorder as functions of the excitation frequency. Such simultaneous recordings, give the possibility of an unambiguous determination of the actual frequencies of vibration of the specimen under investigation. At the same time, by making the recordings in a suitable mesh of panel points, the deflection mode shape corresponding to each frequency could be accurately obtained.

Tests were performed on three stiffened panels made of an aluminum alloy sheet cover stiffened by only one Zed section stringer; the panel dimensions are shown in fig.2. The choice of such types of specimens was made as they embody all those geometric features which are relevant as far as a correct evaluation of the proposed theoretical approach is concerned. At the same time the choice of a single stiffener reduces to a minimum the degrees of freedom (seven) of the vibrating panels making possible the correct excitation of a significant number of vibration modes through the use of the eight shakers which equipped the



PANEL	L -mm-	b -mm-	h -mm-	fa -mm-	f -mm-	t -mm-	ts -mm-
A	600	240	48	35	35	1.6	1.6
B	600	240	48	10	35	1.6	1.6
C	600	240	120	40	40	1.6	1.6

Fig. 2 - Geometry and dimensions of panels tested.

test apparatus.(*)

The specimens A and C were stiffened by a symmetric Zed section stringer and had geometric ratios which are broadly representative of typical aircraft dimensioning. They differed principally in the ratio h/b which has a relevant influence on the phenomenon of the bending-torsion-local interaction; panel A presented a low value of such a ratio, whereas panel C a moderately large value; another significant difference was on the ratio f/h. Panel B was characterized, on the contrary, by an higly asymmetric Zed section stringer with the attached flange of strongly reduced dimension The choice of such dimensioning was dictated by the following two reasons:
- to increase the bending-torsion coupling to the detrement of the coupling with the local mode by reducing the bending-torsional stiffness by an amount which was much larger than the local stiffness and the bending-torsion cross coupling term; such a dimensioning gave an extreme condition, very significant, it is felt, as regards the theoretical approach evaluation.
- to minimize a possible interference between the motions of both the attached flange and the sheet-cover, a phenomenon not allowed in the theoretical approach.

The test procedure was the following one: for each panel tested, it was first controlled that the mode shape in the direction parallel to stringer axis conformed to sinusoidal half waves, to verify the correct fulfilment of the boundary conditions.(**)

Unwanted restraints which might arise because of panel-frame fitting operation, demanded such a type of control.

(*) The choice of eight shakers resulted to be the best solution as far as the compressed air supply characteristic, the amplitude of each exciter force, the drive of all the exciters by a motor alone are concerned.

(**) The achievement of simply supported edges at the panel ends orthogonal to the stringer axis presents some difficulties due to the need of restraining the stringer ends, having respect for the actual stringer displacements which result from the interaction of the bending-torsion and local modes of deformation. A way for fulfilling such boundary conditions was envisaged in ref.[7] and restraint device was accordingly developed which resolved the problem satisfactorily. However the mounting operations require some cautions and therefore it cannot be excluded that unwanted restraints are occasionally imposed to the panel in the course of such operations.

Next the displacement amplitude and the force-displacement phase angle were recorded as functions of the excitation frequency in a set of points chosen in conformity to the expected mode shape. The investigation was limited to the frequency range < 400 Hz as in this field the instrument sensitivity and resolution were adeguate to the amplitude response of the panel.
At the same time the theoretical approach discussed in the previous section was applied to such types of stiffened panels. Five degrees of freedom were allowed for the stringer cross section as shown in fig. 3: the local distortion was taken into account by two rotational degrees of freedom θ_1^L and θ_2^L, and a translational degree of freedom ξ_2^L; bending and torsion by the displacement ξ and the rotation θ^T. The quadratic form which expresses the action for the entire stiffened panel is given in appendix 2.

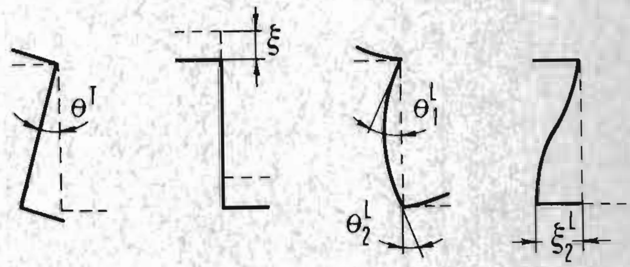


Fig. 3 - Displacements and degrees of freedom for Zed stringer. The actual stringer deflection shape results from the combination of the four sketched shapes.

The corresponding eigenvalue problem was resolved through a computer program which furnishes the frequencies and the vibration mode shapes; the same program gives also the frequencies for both the bending-torsional and local mode of vibration. The main results of such an investigation are collected in tables I, II, III and in the figures 4, 5, 6, 7.

In the first two columns the tables show the experimental values of the resonant frequencies, f_e, and the present approach theoretical values of the natural vibration frequency, $f_{p.a.}$. The subsequent two columns show the natural vibration frequencies computed on the basis of stringer bending-torsion deformation, $f_{B.T.}$, and stringer local deformation, f_L, respectively.

The other columns furnish the present approach theoretical values of quantities which adequately characterize the deflection mode shape: namely the number m_l of longitudinal half-waves, the number m_t of transversal half-waves comprised between the edge and the stringer, the ratios θ_1^L/θ^T, θ_2^L/θ^T, $\xi/(\theta^T b)$, $\xi_2^L/(\theta^T b)$ the meaning of which are clearly deducible from fig. 3.

f_e	$f_{p.a.}$	$f_{B.T.}$	f_L	m_l,m_t	θ_1^L/θ^T	θ_2^L/θ^T	$\xi/(b\theta^T)$	$\xi_2^L/(b\theta^T)$
84	86	92	86	1,1	4.9	2.4	0.05	0.6
101	105	109	-	1,1	-0.9	-0.7	0.11	-0.15
124	125	137	125	2,1	31	-11	-0.06	0.59
136	139	140	-	2,1	-1	-0.6	0.11	-0.05
174	177	187	177	3,1	146	-60	0.05	0.66
183	188	188	-	3,1	-1	-0.7	0.11	0.01
220	229	259	-	1,2	-0.1	0.8	0.17	0.07
246	250	260	250	4,1	400	-153	0.05	0.64
254	260	260	-	4,1	-1	-0.7	0.11	0.0
261	270	-	249	1,2	-2.9	-1.7	0.28	-0.5
325	336	369	336	2,2	26	-10	0.04	0.56
333	338	310	336	1,2	-1.5	7.6	-0.07	0.74

TAB. I - Experimental frequencies and theoretical results for panel A.

f_e	$f_{p.a.}$	$f_{B.T.}$	f_L	m_l,m_t	θ_1^L/θ^T	θ_2^L/θ^T	$\xi/(b\theta^T)$	$\xi_2^L/(b\theta^T)$
84	85	93	90	1,1	1.56	0.62	0.08	0.18
101	105	108	-	1,1	-0.88	-0.19	0.25	-0.09
123	123	136	124	2,1	13.6	-4.8	0.06	0.2
137	138	139	-	2.1	3	-3.8	0.2	0.02
173	176	184	176	3,1	60	-24	0.06	0.21
185	187	188	-	3,1	7.4	-3	0.17	0.025
217	217	226	-	1,2	-0.24	0.68	0.15	0.04
245	249	256	249	4,1	170	-64	0.05	0.2
247	258	258	-	4,1	20.4	-7.8	0.17	0.024
258	259	-	256	1.2	-9	-10	0.8	-2.1
328	343	310	333	1,2	-0.8	-9	0.04	1.27
341	332	360	346	2,2	113	-4.2	0.06	0.19

TAB.II - Experimental frequencies and theoretical results for panel B.

Such a set of data deserves some comments.

The agreement between the values of the frequencies found by test and computed through the present approach (first two columns in the table) is generally fairly good. Only when the stringer cross section distortion strongly depends on the displacement ξ_2^L (see fig. 3) an agreement a little less satisfactory may be found (~10 Hz difference between the experimental and theoretical data).

f_e	$f_{p.a.}$	$f_{B.T.}$	f_L	m_l,m_t	θ_1^L/θ^T	θ_2^L/θ^T	$\xi/(b\theta^T)$	$\xi_2^L/(b\theta^T)$
85	88	108	88	1,1	21	-9	0.05	0.63
106	110	111	-	1,1	0.7	-0.5	0.14	0.02
120	120	136	120	2,1	243	-102	0.04	0.45
137	138	139	-	2,1	23	-10	0.15	0.04
171	172	184	172	3,1	762	-256	0.04	0.34
182	186	186	-	3,1	93	-32	0.15	0.04
247	251	264	-	1,2	2.4	12	0.06	0.15
295	285	-	290	1,2	2.2	6	0.07	- 2.2
310	316	336	316	1,2	1.4	6	0.06	-0.05
333	340	342	-	1,2	-6	38	- 0.4	2.1
375	395	-	308	1,2	-2	-51	-0.1	-0.6

TAB.III- Experimental frequencies and theoretical results for panel C.

In such cases some differences between the experimental and theoretical values are also observed in the mode shape, particularly for panel C, (figs. 7a, 7b), in the zone of the bottom edge of the stringer. Such moderate disagreement may, probably, be ascribed to the difficulty of taking into account accurately the bending deformation of the stringer bottom flange induced by the displacement ξ_2^L owing to the restraints offered to the flange in-plane displacements by the web when such a kind of deformation takes place. In the present approach such an influence was taken into account computing the moment of inertia of the flange as if 20% of the web sheet was effective in resisting to bending deformation. This crude approximation of the actual in-plane stress system in the web due to the degree of freedom ξ_2^L, is, it is felt, the fundamental reason for such disagreement. Further investigations on this topic are now in progress.

Apart from such a secondary effect the adequacy of the proposed approach can be considered proved.

A second comment concerns the fact that the present approach predicts the entire set of frequencies, at least in the range of the experimental investigation. The other two methods, namely the bending-torsion and local approach, furnish on the contrary values of the frequency (3-rd, and 4-th columns) covering only a subset of the actual frequency set. This, truly, is not a surprising fact since both the approaches allow only a reduced number of degrees of freedom with respect to the present approach. In particular the local approach disregards the stringer sheet junction line displacement and therefore cannot predict all those vibration modes where the driving parameter is this displacement. It is, furthermore, worthwhile to observe the agreement between the local approach frequencies and the experimental ones whenever the junction line displacement is small, a fact which makes clear the role of the stringer cross section distorsion in such kinds of phenomena. The bending-torsion approach on the contrary cannot predict the frequencies of those modes which are excited prevailingly by stringer cross section distortion. Figs. 5c, 6c show for instance typical examples

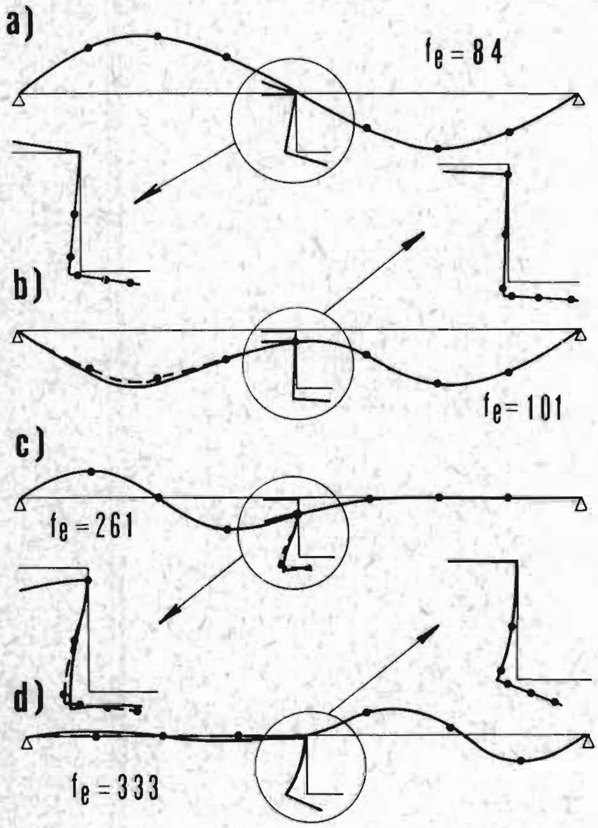


Fig.4 - Experimental and theoretical mode shapes for panel A; for each mode shape experimental value of frequency is given. Through this quantity with the aid of the table I the theoretical values of the generalized coordinates can be obtained.

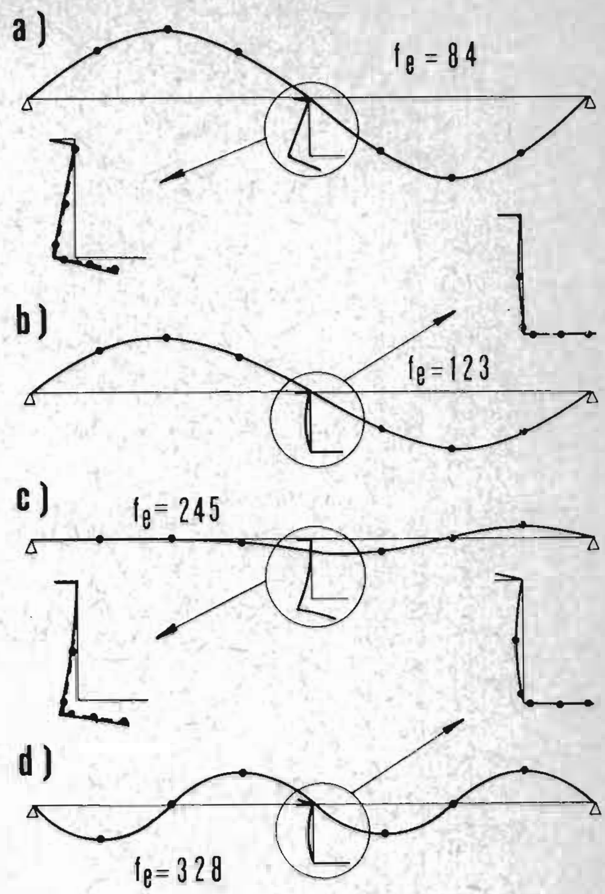


Fig. 5- Experimental and theoretical mode shapes for panel B; for each mode shape , experimental value of frequency is given. Through this quantity with the aid of the table II the theoretical values of the generalized coordinates can be obtained.

of such modes; tables I, II, III, clarify what frequencies are lost through such an approach.

It is therefore surprising that such a method which suffers from not negligible limitations, continues to be used in the field of acoustic fatigue [8]. The inadequacy of the bending-torsion approach results still greater if the deflection mode shapes are considered. The last four columns of tables I, II, III show the generalized coordinates of the stringer cross section normalized through the torsional rotation θ^T. It is worthwhile to note the values of the quantities θ_1^L, θ_2^L, ξ_2^L which completely describe stringer cross section local deformation; they are generally not negligible and in several cases largely dominant on the other generalized coordinates. As a local deformation implies biaxial stress systems, linearly varying through the wall thick - ness, which can assume large values with relatively small local deflection, the im- portance of the correct assessment of lo - cal distortion due to vibration results clear in design against acoustic fatigue , at least when formed stringers are used as stiffeners.

The importance of the cross section distortion on the vibratory behaviour of such panels clearly results also from the examination of figs 4, 5, 6, 7 which show the panel cross section mode shapes as ob- tained by test compared with the present approach theoretical modes(*).

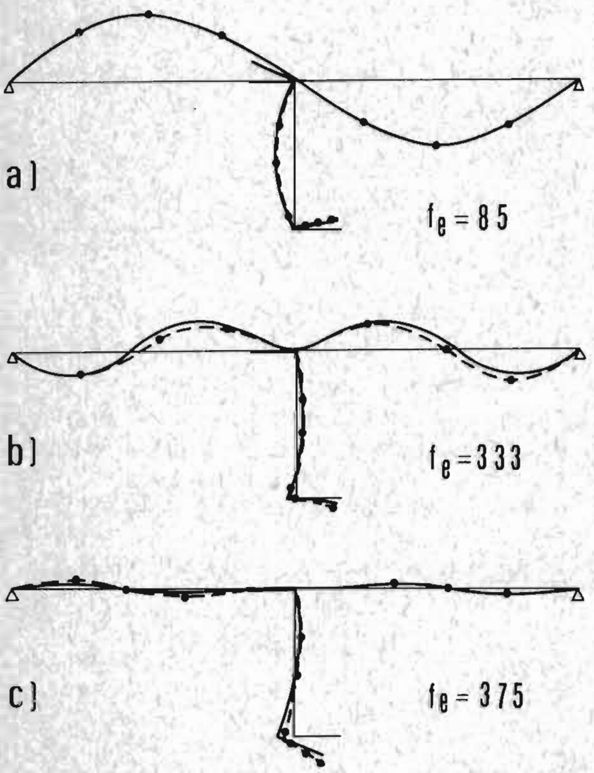


Fig.6 - Experimental and theoretical mode shapes for panel C; for each mode shape experimental value of frequency is given. Through this quantity with the aid of the table III, the theoretical values of the generalized coordinates can be obtained.

(*) The theoretical mode shape amplitude is defined apart from a moltiplicative con stant which in the present comparison was determined fitting by eye the theoretical curve with the experimental data; other more involved fitting procedures, in prin-

Each figure is relative to a panel and indicates for each mode the experimental frequency by which, with the aid of tables I, II, III all the theoretical relevant quantities which characterize the mode shape can be obtained. Because of report format requirements, only a limited a- mount(**) of the measured and computed mode shapes are shown; with the exception of figs 7a, 7b the reported data are, howev - er, quite representative of all the other mode shapes obtained in the course of the present investigation, as far as the agree ment between theoretical and experimental data is concerned, and therefore they can be confidently used to drawn conclusions about the validity of the proposed ap - proach. The modes of fig. 7a and 7b on the contrary were the only, among the 35 re - corded, which presented some discrepancies with respect to the theoretical values.

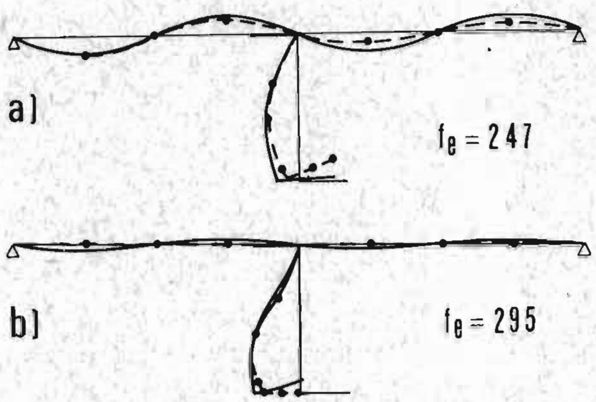


Fig.7 - Results for the 7-th and 8-th modes of panel C. The differences found in the stringer deformations were the unique ex- amples of disagreement worth quoting.

The reason for the localized disagree- ment around the stringer bottom edge, as already noted, may probably be ascribed to the stress system in the stringer web due to flange bending, allowed in an approxi- mate manner in the present approach.

Two general comments are worthwhile:the adequacy of the test procedure, and the

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(*) ciple more rational, would result time consuming without improving substantially the comparison .

(**) The complete collection of data will be published as an Institute of Aeronau - tics Report, University of Pisa.

excellent agreement, apart from the two discussed cases, between the theoretical and experimental results. The first fact clearly results from fairly good excitation of all the modes falling in the range of frequency of investigation. The second observation is grounded on the successful comparison of 35 modes found both experimentally and theoretically. Such a successful comparison represents an undoubtedly sound verification of the present approach in determining not only the frequencies but also the modes of vibration and all the other related quantities such as the stress systems.

Other more particular comments deserve to be quoted; as far as the influence of the geometry the increasing importance of the local deformation with the increase of the stringer-web dimension can be observed.

This fact already clearly results from the examination of the first mode which is prevailingly torsional for panels A, B and prevailingly local for panel C. With regard to the influence of the stringer bottom flange two distinct trends can be observed: a low width flange promotes modes of vibration in the lower frequency range where the stringer cross section distortion depends prevailingly on the displacement ξ_2^L ; a large width flange gives rise to stringer cross section distortion prevailingly in the degree of freedom θ_2^L.

The local deformation, besides, becomes more and more important with the decreasing of the longitudinal wave length owing to the strong increase in bending-torsion stiffness of the stringer.

At least an observation can be drawn from the comparison of the behaviour of panels A and B as far as the interaction between flange and sheet-cover is concerned. Such a comparison seems to indicate that such an effects is really negligible.

IV. CONCLUSIONS

A research on the frequencies and modes of vibration of stiffened panels has been carried out in the present investigation. A theoretical approach based on Hamilton's principle, was at first developed for computing the frequencies and the modes of vibration of such panels. The distinctive aspect of such an approach is a description of the stringer deformation which takes into account both bending-torsion displacement and cross section distortion.

A general procedure has been outlined to obtain the action for a given stiffened panel in the case in which the stringer cross section distortion can be allowed only through rotational degrees of freedom; the case has also been treated, in which some translational degrees of freedom play a not negligible role in the stringer cross section distortion. Such an approach was applied to the case of a panel stiffened by one Zed section stringer of various dimensions, the frequencies and the corresponding modes were obtained through a suitable computer program .

At the same time tests were performed on three panels stiffened by one Zed section stringer by a test facility which utilizes pneumatic exciters and displacement non-contacting transducers. With the aid of such facilities all the frequencies and modes falling in the range of frequency < 400 Hz were recorded.

The data obtained were compared with the theoretical results. The basic considerations coming from such a comparison can be summarized as follows:

- The cross section distortion of the stringer plays an important role in the vibrations of the stiffened panels considered in the present investigation. In particular the prediction of the panel stress system through a modal analysis is quite inaccurate if the above-said distortion is ignored in mode shape evaluation.
- The present theoretical approach gives the possibility of an accurate description of the stringer deformation allowing at the same time stringer bending-torsion displacement and cross section distortion. The procedure outlined is sufficiently general and easily applicable to a given panel configuration.
- The experimental results in terms both of frequency and mode shape, obtained in the course of the present investigation, are, apart from some minor discrepancies, in good agreement with the theoretical results and demonstrate the soundness of the proposed theoretical approach.

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APPENDIX 1

The analytical expressions of the four basic deflection shapes shown in fig. 1 are the following ones:

$$f_i^s = \frac{-\alpha_2 \operatorname{senh}(\alpha_2/2)\cosh(\alpha_1\eta)+\alpha_1 \operatorname{senh}(\alpha_1/2)\cosh(\alpha_2\eta)}{\alpha_1 \operatorname{senh}(\alpha_1/2)\cosh(\alpha_2/2)-\alpha_2 \operatorname{senh}(\alpha_2/2)\cosh(\alpha_1/2)}$$

$$f_i^a = \frac{\alpha_2 \cosh(\alpha_2/2)\operatorname{senh}(\alpha_1\eta)-\alpha_1 \cosh(\alpha_1/2)\operatorname{senh}(\alpha_2\eta)}{\alpha_1 \cosh(\alpha_1/2)\operatorname{senh}(\alpha_2/2)-\alpha_2 \cosh(\alpha_2/2)\operatorname{senh}(\alpha_1/2)}$$

$$\phi_i^s = b_i \frac{-\cosh(\alpha_2/2)\cosh(\alpha_1\eta)+\cosh(\alpha_1/2)\cosh(\alpha_2\eta)}{\alpha_1 \operatorname{senh}(\alpha_1/2)\cosh(\alpha_2/2)-\alpha_2 \operatorname{senh}(\alpha_2/2)\cosh(\alpha_1/2)}$$

$$\phi_i^a = b_i \frac{\operatorname{senh}(\alpha_2/2)\operatorname{senh}(\alpha_1\eta)-\operatorname{senh}(\alpha_1/2)\operatorname{senh}(\alpha_2\eta)}{\alpha_1 \cosh(\alpha_1/2)\operatorname{senh}(\alpha_2/2)-\alpha_2 \cosh(\alpha_2/2)\operatorname{senh}(\alpha_1/2)} \qquad (1-1)$$

The stiffness relative at the previous cases are:

$$\mu_i^s = \frac{Et_i^3}{12(1-\nu^2)}\,\frac{1}{b_i}\,\frac{2(\omega/\omega_i^*)}{\alpha_1 \operatorname{tgh}(\alpha_1/2)-\alpha_2 \operatorname{tgh}(\alpha_2/2)}$$

$$\mu_i^a = \frac{Et_i^3}{12(1-\nu^2)} \frac{1}{b_i} \frac{2(\omega/\omega_i^*)}{\alpha_1 cotgh(\alpha_1/2) - \alpha_2 cotgh(\alpha_2/2)}$$

$$k_i^s = \frac{Et_i^3}{12(1-\nu^2)} \frac{1}{b_i^3} \frac{2(\omega/\omega_i^*)}{cotgh(\alpha_2/2)/\alpha_2 - cotgh(\alpha_1/2)/\alpha_1}$$

$$k_i^a = \frac{Et_i^3}{12(1-\nu^2)} \frac{1}{b_i^3} \frac{2(\omega/\omega_i^*)}{tgh(\alpha_2/2)/\alpha_2 - tgh(\alpha_1/2)/\alpha_1}$$

$$(\mu k)_i^s = (k\mu)_i^s = \frac{Et_i^3}{12(1-\nu^2)} \frac{1}{b_i^2} \left\{ \frac{\left[\omega/\omega_i^* - (1-\nu)(\frac{\pi}{\lambda} b_i)^2\right]\alpha_1 \; senh(\alpha_1/2)\cosh(\alpha_2/2)}{\alpha_1 senh(\alpha_1/2)\cosh(\alpha_2/2) - \alpha_2 senh(\alpha_2/2)\cosh(\alpha_1/2)} + \right.$$

$$\left. + \frac{\left[\omega/\omega_i^* + (1-\nu)(\frac{\pi}{\lambda} b_i)^2\right]\alpha_2 senh(\alpha_2/2)\cosh(\alpha_1/2)}{\alpha_1 senh(\alpha_1/2)\cosh(\alpha_2/2) - \alpha_2 senh(\alpha_2/2)\cosh(\alpha_1/2)} \right\}$$

$$(\mu k)_i^a = (k\mu)_i^a = \frac{Et_i^3}{12(1-\nu^2)} \frac{1}{b_i^2} \left\{ \frac{\left[\omega/\omega_i^* - (1-\nu)(\frac{\pi}{\lambda} b_i)^2\right]\alpha_1 \; \cosh(\alpha_1/2) senh(\alpha_2/2)}{\alpha_1 \cosh(\alpha_1/2) senh(\alpha_2/2) - \alpha_2 \cosh(\alpha_2/2) senh(\alpha_1/2)} + \right.$$

$$\left. + \frac{\left[\omega/\omega_i^* + (1-\nu)(\frac{\pi}{\lambda} b_i)^2\right]\alpha_2 \cosh(\alpha_2/2) senh(\alpha_1/2)}{\alpha_1 \cosh(\alpha_1/2) senh(\alpha_2/2) - \alpha_2 \cosh(\alpha_2/2) senh(\alpha_2/2)} \right\} \qquad (2.1)$$

For a flange, namely a strip, with one of edges parallel to stringer axis free to translate and rotate, the other simply supported and elastically built-in the cross-section deflection shape ϕ^F and the rotational stiffness μ^F can be written in the following form:

$$\phi^F = \frac{\left[\alpha_1 P^2 \cosh(\alpha_1) senh(\alpha_2) - \alpha_2 Q^2 \cosh(\alpha_2) senh(\alpha_1)\right]\left[\cosh(\alpha_1 \eta) - \cosh(\alpha_2 \eta)\right]}{\alpha_1 \alpha_2 \left[2PQ + (P^2+Q^2)\cosh(\alpha_1)\cosh(\alpha_2) - Rsenh(\alpha_1) senh(\alpha_2)\right]} +$$

$$+ \frac{\left[\alpha_2 Q^2 \cosh(\alpha_1)\cosh(\alpha_2) - \alpha_1 P^2 senh(\alpha_1) senh(\alpha_2) + \alpha_2 PQ\right] senh(\alpha_1 \eta)}{\alpha_1 \alpha_2 \left[2PQ + (P^2+Q^2)\cosh(\alpha_1)\cosh(\alpha_2) - Rsenh(\alpha_1) senh(\alpha_2)\right]} +$$

$$+ \frac{\left[\alpha_1 P^2 \cosh(\alpha_1)\cosh(\alpha_2) - \alpha_2 Q^2 senh(\alpha_1) senh(\alpha_2) + \alpha_1 PQ\right] senh(\alpha_2 \eta)}{\alpha_1 \alpha_2 \left[2PQ + (P^2+Q^2)\cosh(\alpha_1)\cosh(\alpha_2) - Rsenh(\alpha_1) senh(\alpha_2)\right]} \; ;$$

$$\mu^F = \frac{Et_i^3}{12(1-\nu^2)} \frac{\omega/\omega_1^*}{b_i \alpha_1 \alpha_2} \frac{Q^2 \alpha_2 \cosh(\alpha_2) senh(\alpha_1) - P^2 \alpha_1 \cosh(\alpha_1) senh(\alpha_2)}{\left[2PQ + (P^2+Q^2)\cosh(\alpha_1)\cosh(\alpha_2) - Rsenh(\alpha_1) senh(\alpha_2)\right]} \qquad (3\text{-}1)$$

where $P = \omega/\omega_i^* - (\frac{\pi b_i}{\lambda})^2 (1-\nu)$; $Q = \omega/\omega_i^* + (\frac{\pi b_i}{\lambda})(1-\nu)$;

(1-); $R = \left[\alpha_1 P^2/\alpha_2 + \alpha_2 Q^2/\alpha_1\right]$

The axis $y = \eta b_i$ has origin at the restrained edge and is directed toward the free edge.

APPENDIX 2

In the case of a panel stiffened by one Zed section stringer with the deformation pattern sketched in fig.3, the action can be written applying the relationship (8) with the some modifications wich depend on the translational degree of freedom ξ_2^L. In particular $\mu^L(\theta^L)^2$ must be replaced by the following expression:

$$\mu^L(\theta^L)^2 = \mu^s_{1,2}\left(\frac{\theta_1^L-\theta_2^L}{2}\right)^2 + \mu^a_{1,2}\left(\frac{\theta_1^L+\theta_2^L}{2}\right)^2 + \mu_1^F(\theta_1^L)^2 + \mu_2^F(\theta_2^{L})^2 + k^s_{1,2}\left(\frac{\xi_2^L}{2}\right)^2 + k^a_{1,2}\left(\frac{-\xi_2^L}{2}\right)^2 + 2(\mu k)^s_{1,2}\left(\frac{\theta_1^L-\theta_2^L}{2}\right)\left(\frac{\xi_2^L}{2}\right) + 2(\mu k)^a_{1,2}\left(\frac{\theta_1^L+\theta_2^L}{2}\right)\left(\frac{-\xi_2^L}{2}\right) + F(\xi_2^L)^2 \qquad (1.2)$$

where the notation of fig.3 has been used; here F is the generalized stiffness of the bottom flange due to the displacement ξ_2^L. Further a new cross coupling term must be added to the ones given by the relation - ship (7),(7') ;which takes into account the interaction between the two degrees of freedom θ^T, ξ_2^L. Such cross coupling term can be computed on the basis of the reasoning of the section II.

DISCUSSION

I. Elishakoff (Dept. of Mechanics, Technion I.I.T., Haifa, Israel): 1. Did you take into account the finite width of the stiffener? i.e. was the stiffener a line stiffener only, or not?
2. The plate considered in your paper was simply supported in z-direction. This means that the problem could be solved exactly as Professor Y.K. Lin and his followers did.

G. Cavallini and A. Salvetti : 1. Yes, we took into account the finiteness of the stiffener's width. In fact the stiffener deformation resulted, as is shown in Fig. 3, from rigid displacements (bending-torsion deformation) and distortion of the stiffener cross-section. To obtain an accurate description of such a distortion the stiffener is considered as an assembly of component flats which behave as plates.
2. The fundamental difference between the present approach and the one of Prof. Y.K. Lin lies in the type of stiffener deformation that we have taken into account in our analysis. The boundary conditions of the stiffener-sheet cover junction lines which must be specified in the Lin approach depend on the mode of deformation of the stiffener which results from a combination of bending torsion and local distortion which is unknown "a priori". In this condition the variational approach is a really rational procedure to solve the problem in a correct and general way. Further the present approach is "exact" as it embodies all the implications of the actual boundary conditions in the z-direction.

Apart from the procedure to solve the problem it seems important to emphasize, as a further comment to our investigation, that the local distortion plays an important role in the vibration of the panel and this is an important matter as far as acoustic fatigue is concerned.